

Summer School
“Singularities in Science and Engineering”
Ghent University, 22-31 Aug. 2022

FRACTIONAL DERIVATIVE IN SCIENCE
AND ENGINEERING:

Chronicles and Perspectives of FC,

In Memory of JAT Machado †

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1. TRIBUTE TO MEMORY OF PROF. JAT MACHADO

– DON QUIXOTE of FC

May God bless his soul !

On behalf of the Editorial Board of “Fractional Calculus and Applied Analysis” and all Fractional Calculus community, we need to share our sincere sadness for the great loss that the sudden demise of Professor Jose Antonio Tenreiro Machado caused in autumn of 2021.

We lost a never tired and ever fighting Don Quixote of the FC, its great promoter and so active researcher,

Associate Editor in FCAA, Editor in many other important publications, Organizer of several FC meetings, and Author of numerous works promoting and explaining the applications of FC.

Till his last breath he was initiating papers, books, events and special issues, served to many journals, chaired a conference session (even on his last day), full of energy and life!

Prof. J. A. Tenreiro Machado, 1957-2021



October 6, 1957 - October 6, 2021

His life's motto

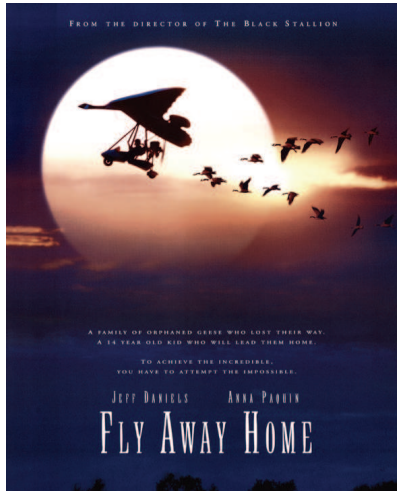
Hilbert's PhD oath

"I ask you solemnly whether by the given oath you undertake to promise and confirm most conscientiously that you will defend in a manly way true science, extend and embellish it, not for gain's sake or for attaining a vain shine of glory, but in order that the light of God's truth shine bright and expand."

These 3 slides were provided by Prof. Alexandra Galhano,
his widow

Prof. J. A. Tenreiro Machado, 1957-2021

His last moments



Prof. Machado's most important scientific production is in the area of FC. His first paper on the subject [1] dates back to 1997. From then on, he worked and published several papers on FC, of which 51 as a single author. In 2019, he edited a multi-volume book series entitled "*Handbook of Fractional Calculus with Applications*" with a total of 8 volumes, [2]. Among his scientific production, we can find works magnificent for their insight and elegance, [3]-[7].

Professor Machado's contributions to the FC community, as author, editor, one of the most successful and energetic promotor of FC and organizer of events and round table discussions on the topic, can hardly be listed in short. Among these activities, let us mention his [hostage and organization of the 2nd International Conference "Fractional Differentiation and Applications, Porto 2006"](#) (now regular biannual conferences ICFDA), and his role as member of Editorial Board and recently, as [Associate Editor in FCAA journal](#). His last paper in FCAA, that can serve as a tutorial in education on FC, appeared just recently in end of 2021, [8].

References

[1] J. A. T. Machado, Analysis and design of fractional-order digital control systems. *Systems Analysis Modelling Simulation* 27, No 2-3 (1997), 107–122; [2] J. A. T. Machado, Ed., *Handbook of Fractional Calculus with Applications*, Vos. 1–8. De Gruyter (2019); [3] J. A. T. Machado, Fractional order generalized information. *Entropy* 16 (2014), 2350–2361; [4] J. A. T. Machado, Complex evolution of a multi-particle system. *Applied Mathematical Modelling* 37, No 22 (2013), 9203–9214; [5] J. A. T. Machado, Entropy analysis of systems exhibiting negative probabilities. *Communications in Nonlinear Science and Numerical Simulation* 36 (2016), 58–64; [6] J. A. T. Machado, Bond graph and memristor approach to DNA analysis. *Nonlinear Dynamics* 88, No 2 (2017), 1051–1057; [7] J. A. T. Machado, Mathematical and computational modeling of political systems. *Nonlinear Dynamics* 96, No 2 (2019), 1471–1490; [8] J. A. Tenreiro Machado, The bouncing ball and the Grünwald-Letnikov definition of fractional derivative. *Fract. Calc. Appl. Anal.* 24, No 4 (2021), 1003–1014; DOI: 10.1515/fca-2021-0043.

Our community owes to Prof. Machado, along with his scientific results and many surveys on history and science metrics in FC, [several very popular posters](#), such as “A poster about the old history of Fractional Calculus”, “A poster about the recent history of Fractional Calculus” (2010), “Fractional Calculus – Models, Algorithms, Technology” (2015), etc.

As a personality, Tenreiro had very wide interests, including the hobby to illustrate science and FC in [his specific artistic ways, hinted at from famous movies, novels, evergreens, paintings, ...](#) such as: “Mission: Impossible” (television and film series) $\Rightarrow$ “FC: Mission impossible“, “Alice’s Adventures in Wonderland” (L. Carroll) $\Rightarrow$ “FC adventures in Wonderland”, “Rhapsody in Blue” (G. Gershwin) $\Rightarrow$ “Rhapsody in fractional”, “What a wonderful world” (L. Armstrong) $\Rightarrow$ “And I say to myself: What a fractional world!”, “D’où venons-nous? Que sommes-nous? Où allons-nous?” (P. Gauguin), [“The persistence of memory”](#) (S. Dali), or “FC: Quo Vadimus?”.

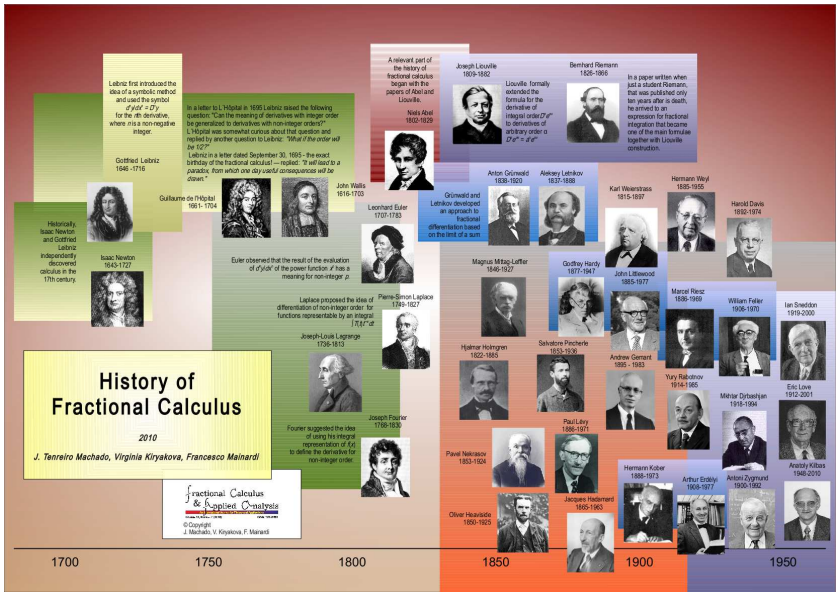


Figure: The time line of fractional calculus during the period 1695-1970

Many of Machado's posters and presentations, their titles and notes for round table discussions of FC conferences, published in FCAA and elsewhere, were inspired from arts and literature, among which:

"Suggestion from the past?", "And I say to myself 'What a fractional world!' ", "Rhapsody in fractional", "Fractional calculus: Quo vadimus? (Where are we going?)", "Fractional calculus: D'où venons-nous? Que sommes-nous? Où allons-nous?", "Fractional calculus's adventures in wonderland", etc.

Aside from such replicas inspired from arts and humanity, one of his picturesque and interesting [presentation is on applications of FC in Science, Engineering and Industry](#), It was a 45 pages lecture of Prof. Machado at one of our Conferences in Sofia, 2017. There he made references of FC ideas as to: Pyramids, Aircrafts, Breakwaters, Electromagnetic diffusion, CRONE models for Cars industry and Road traffic, Lung inpedance, Fractance, Botanics (Plants), Wines, Terrorist attacks, Soccer dynamics, etc.

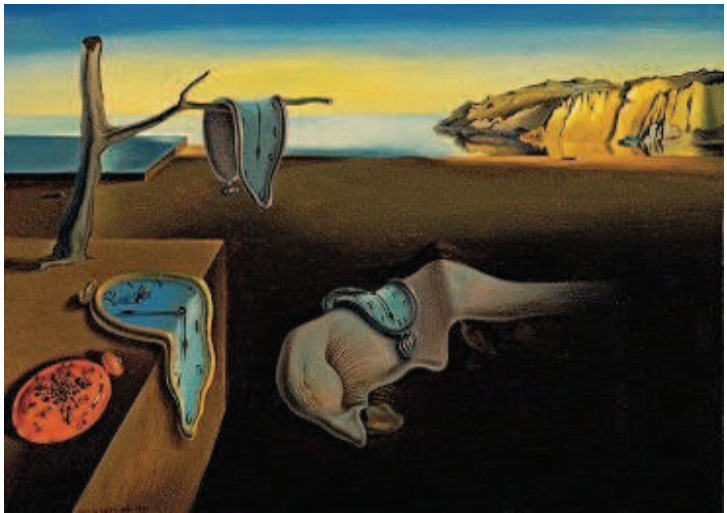
Fractional Calculus: Fundamentals and Applications

"Transform Methods and Special Functions 2017", 8th International
Conference, Sofia, August 27-31, 2017

J. Tenreiro Machado

Institute of Engineering of Porto, Portugal

The file of this presentation is available at request by participants



Salvador Dalí: The persistence of memory

Surely, FC community will keep strong “Persistence of memory”
toward the personality of the “fractional calculus” man,
JAT Machado . . .

2. CHRONICLES OF FRACTIONAL CALCULUS

In years 2010-2020, we had a very fruitful collaboration with Prof. Machado on preparation and publications of several SURVEY PAPERS on history and recent development of the FC and its applications, aside from the above Posters.

Also, notes from Round Tables discussions at ICFDA conferences, on problems of the FC progress, were co-authored by JAT Machado - V. Kiryakova - F. Mainardi. They have been widely cited in subsequent works on FC by other authors.

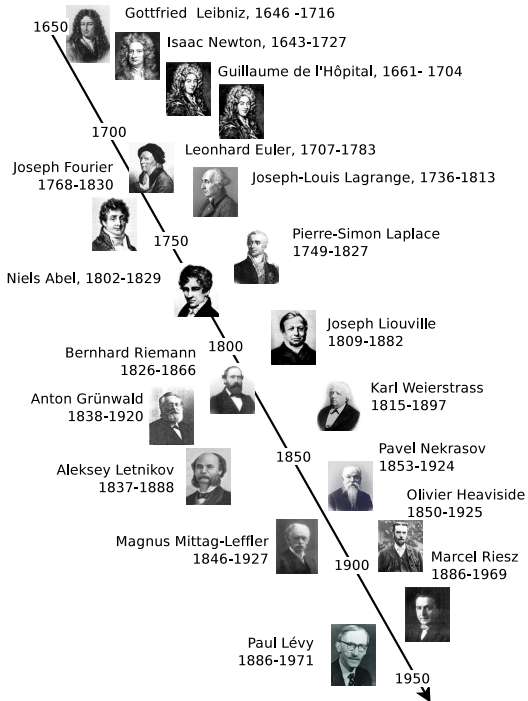
We will present briefly some main information from these joint works.

The latest one, J.A. Tenreiro Machado and V. Kiryakova: "Recent history of the fractional calculus: data and statistics" is published in Vol. 1 of 8 volumes' "*Handbook of Fractional Calculus with Applications*", De Gruyter (2019), initiated and edited by Prof. Machado. (Also in: *FCAA*, Vol. 20, No 2 (2017), 307-336; DOI: 10.1515/fca-2017-0017.)

In the year 1695 in a letter from l'Hôpital to Leibniz the question was raised if the derivative of integer order could be extended to fractional values, say $1/2$. Leibniz studied the problem for the power function and replied that *"It leads to a paradox, from which one day useful consequences will be drawn."* During the next centuries the topic was further developed, but long it remained in the limited scope of pure mathematics as an exotic theory.

However, *two remarkable events happened simultaneously in 1974*. The first conference dedicated to FC was due to Bertram Ross who organized the International Conference on Fractional Calculus and its Applications at the University of New Haven in 1974, and who edited its proceedings (as *L.N.M.*, 1975). The same year it appeared also the first monograph, authored by Keith B. Oldham (a chemist) and Jerome Spanier (mathematician): *"The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order"*, Academic Press, New York (1974).

In the survey we provided a long list of *some of the pioneers of FC* of that (initial) era.



An evidence for the wide spread of FC as a tool in mathematical disciplines and other related areas and applications is the following. Since 2010, the Mathematics Subject Classification (MSC 2010, 2020) included not only the basic index 26A33, but many other positions related to FC, like 28A80, 33C60, 33E12, 33E30, 34A08, 34K37, 35R11, 44A20, 44A40, 45E10, 60G22, 93B99, 93D05, etc.

Many of the open problems and challenging hypotheses from the early years (20th century) of the FC development (posed, for example, at 1st and some of next conferences on FC) are nowadays solved or already out of question, at all. But their discussion stimulated researchers and also raised the need of various specialized forums (journals, round tables, meetings).

One of the challenging open problems, risen yet by B. Ross at the 1st FC conference was about any possible geometric of physical interpretations of the operators of FC. Happily, now this is fully out-of-date ... At Tokyo's 1989 conference, also a possible relation between "FRACTional" calculus and "FRACtal" geometry was mentioned, but then apriori denied ...

Transform Methods & Special Functions, Varna'96

Proceedings of Second International Workshop, 23 - 30 August 1996

A LONG STANDING CONJECTURE FAILED?

V. Kiryakova

Convenor of the Round Table Discussion

In the frames of the Second International Workshop "Transform Methods & Special Functions, Varna'96" a round table discussion **"Physical and Geometrical Meanings and Applications of the Fractional Calculus Operators"** took place. Provoked by some open problems in the field of Fractional Calculus, it was well attended and lasted more than 2 hours. *Among the participants were:*

I. Ali, B. Bonilla, P. Butzer, N. Bozhinov, H.-J. Glaeske, R. Gorenflo, I. Dimovski, E. M. Elabd, S. L. Kalla, S. Kempfle, A. Kilbas, Yong Chan Kim, V. Kiryakova, P. Konovski, F. Mainardi, W. Majcharzak, S. Owa, I. Podlubny, M. Rivero, P. Rusev, M. Saigo, H. Saitoh, B. Al-Saqabi, H. M. Srivastava, J. Stankiewicz, R. Tremblay, J. Trujillo, Vu Kim Tuan, A. Al-Zamel, etc.

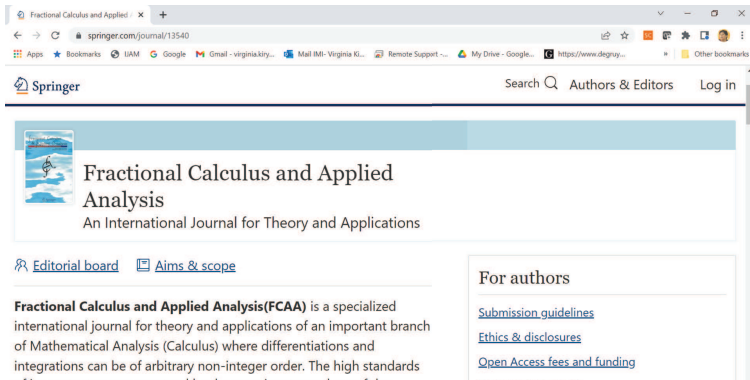
Topics of discussion were, along with the very recent practical applications, *some open problems raised subsequently on the three international conferences* devoted especially to Fractional Calculus, that took place in: Univ. of New Haven (USA) in 1974; Ross Priory, Univ. of Strathclyde - Scotland (UK) in 1984; Nihon University, Tokyo (Japan) - in occasion of its 100th Anniversary in 1989. Each of the Proceedings of these meetings, [1],[2],[3], was ended by a section of such open problems and especially, the following pessimistic conjecture was repeated there, *originally stated by the late Prof. B. Ross*, initiator of the first such conference.

CONJECTURE. Is there a geometrical representation of a fractional derivative? If not, can one prove that a graphical representation of a fractional derivative does NOT exist? ... The consensus of the experts ... is that there is, in general, NO geometrical interpretation of a derivative of fractional order ... It can be asked, however at least for a geometrical meaning or a physical phenomena that can be represented by means of equations involving a derivative of a particular order such as $1/2$...

A suggestion that the *negative parts of the above conjecture might be wrong*, is given by the recent largely exploited notions of **"Fractal Hausdorff Dimensions"** and **"Fractal Geometry"**.

At this conference the above mentioned problems were discussed, and
it was decided that the FC community will need a periodical publication,
a forum, as this appeared next year after “TMSF” Proceedings...
⇒ the *FCAA* journal !

And so, since **Vol. 1 (1998)** we have now **Vol. 25 (2022)**,
published currently by Springer, the 8th in Top10 Rank list of WoS
for Math.: <https://link.springer.com/journal/13540>



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**GEOMETRIC AND PHYSICAL INTERPRETATION
OF FRACTIONAL INTEGRATION AND
FRACTIONAL DIFFERENTIATION ***

Igor Podlubny

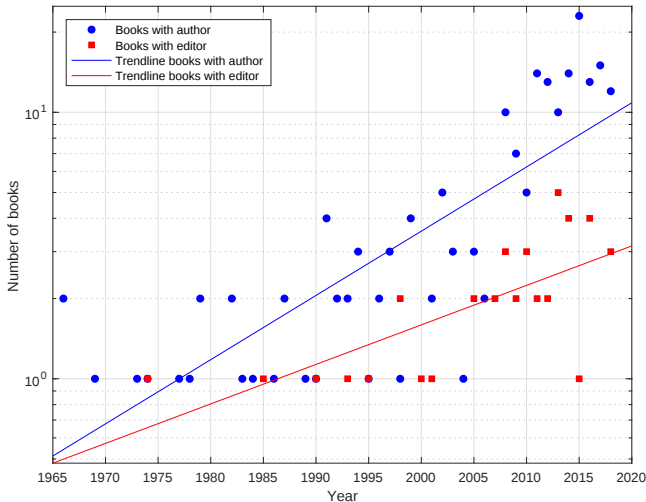
*Dedicated to Professor Francesco Mainardi,
on the occasion of his 60-th birthday*

Abstract

A solution to the more than 300-years old problem of geometric and physical interpretation of fractional integration and differentiation (i.e., integration and differentiation of an arbitrary real order) is suggested for the Riemann-Liouville fractional integration and differentiation, the Caputo fractional differentiation, the Riesz potential, and the Feller potential. It is also generalized for giving a new geometric and physical interpretation of more general convolution integrals of the Volterra type. Besides this, a new physical interpretation is suggested for the Stieltjes integral.

Mathematics Subject Classification: 26A33, 26A42, 83C99, 44A35, 45D05

Key Words and Phrases: fractional derivative, fractional integral, fractional calculus, geometric interpretation, physical interpretation



Number N of **books with author (more than 200)**
and with editor (more than 50)
 versus year x , and trends of the type $\ln N = a + bx$, $a, b \in \mathbb{R}$
 during the period 1965–2018

Journals: *JFC, FCAA, JFCA, PFD, Fract.&Fract.,*

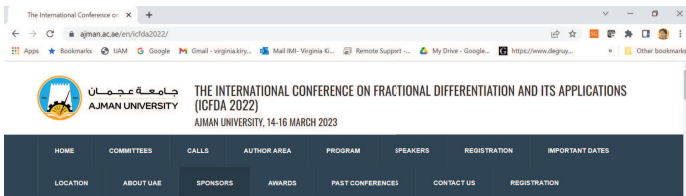
Special and topical issues:

Conferences, FC days, Summer Schools, etc. devoted to FC:
(see Section 3 in the Survey paper Machado-Kiryakova)

Among them, as a traditional events in every 2 years, are:

International Conferences of Fractional Differentiation and Applications (ICFDA)

1st one: Bordeaux 2004 / / 9th will be in March 2023, UAE,
website: <https://www.ajman.ac.ae/en/icfda2022>,
and we are eager for the jubilee 10th one (20 years after 1st):
planned in Bordeaux, 2024



At these ICFDA conferences, as a tradition, Round Tables (RT) discussions took place. Most of them were co-organized by Machado-Kiryakova-Mainardi and by the local hosts. As a result, there appeared a series of notes of these RT, published in FCAA journal, on current problems and perspectives for FC.

One of them, among other questions, presented a view on the time scales in FC models, inspired (again it was a hint by Machado) by a Gauguin's famous picture ...



Paul Gauguin (1897-1898): “Where Do We Come From? What Are We? Where Are We Going?”

Namely, it was a RT at ICFDA 2016 (Novi Sad): “ Fractional Calculus: D'o' u venons-nous? Que sommes-nous? O' u allons-nous?”

Aside from the 3 fundamental questions (in theory known as “catechism”), for the artist the 3 major figure groups were to illustrate the stages of the life (from childhood to old age), when the painting is read from the right to the left ...

As for possible interpretation in view of FC, consider the Liouville's integral of fractional order $\alpha > 0$:

$$I^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{\infty} (x-t)^{\alpha-1} f(t) dt,$$

as

$$I^\alpha f(x) = \int_{-\infty}^{\infty} \dots = \int_{-\infty}^{t_0} \dots + \int_{t_0}^x \dots + \int_x^T \dots + \int_T^{\infty} \dots := I_1 + I_2 + I_3 + I_4.$$

and interpret each part of interval $(-\infty, +\infty)$ as follows.



Time scale in FC (to be read from right to left):

$$t \rightarrow \infty, \quad t = T, \quad t = x, \quad t = t_0 \quad (t = 0), \quad t \rightarrow -\infty$$

$$I_1 = I_+^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{t_0} (x - t)^{\alpha-1} f(t) dt \quad (\text{once upon a time...}),$$

$$I_2 = \frac{1}{\Gamma(\alpha)} \int_{t_0}^x (x - t)^{\alpha-1} f(t) dt, \quad I_3 = \frac{1}{\Gamma(\alpha)} \int_x^T (t - x)^{\alpha-1} f(t) dt$$

(what happens today / today is yesterday's tomorrow...),

$$I_4 = I_-^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_T^\infty (t - x)^{\alpha-1} f(t) dt$$

(whatever will be, will be (que sera, sera...)).

Alice's questions and FC ?



The Cheshire Cat and Alice:
Alice'S Adventures in Wonderland,
by Lewis Carroll

“The destination is the Key”:

Remind a talk where the Cheshire Cat (C.) warns Alice (A.) to make a cautious choice:

- A. “Would you please tell me which way to go from here?” ,
- C. “That depends on where you want to get to” ,
- A. “I don’t much care where ...” ,
- C. “Then it doesn’t matter which way you go” ,
- A. “So long as I get *somewhere...*” ,
- C. “Oh, you’re sure to do that” , said the Cat, “if you only walk *long enough*” .

To follow a right destination (in FC and in Science, in general), one may need to pose some not so pleasant questions, as:

- Does everything in Maths can and need to be fractalized ?
 - Does the great flood of recently published works on FC contributes to its progress, or may damage the impact gained with efforts of several generations ?
- They are observed so many publications on “the hot topic of FC“ of weak quality or simply not adequate for FC, and claiming on strange areas of applications.

3. WARNING ON SOME PROBLEMS IN RECENT FC TRENDS

M.D. Ortigueira, J.A. Tenreiro Machado, What is a fractional derivative? *J. Comput. Phys.* **293**, 4-13 (2015); doi: 10.1016/j.jcp.2014.07.019

– **LOCAL FC**. Some authors claim to introduce the so-called local operators of FC ... Example: Yang Xiaojun, *Advanced Local Fractional Calculus and Its Applications* (2012), and next ones ...

The main essence in **the true FC is that its operators are NONLOCAL**, in contrary to the classical differentiations of 1st and integer order.

In this relation, several critical works have shown the inconsistencies of such theory, some of them as:

V. Tarasov, Local fractional derivatives of differentiable functions are integer-order derivatives or zero. *Intern. J. Appl. and Comput. Math.* **2**, 195-201 (2016); doi: 10.1007/s40819-015-0054-6

V. Tarasov, No nonlocality. No fractional derivative. *Commun. Nonlin. Sci. Numer. Simul.* **62**, 157-163 (2018); doi: 10.1016/j.cnsns.2018.02.019

– **JUMARIE Modified FD** (2006, 2009).

Non-integer order derivatives **cannot satisfy the Leibniz rule** ...

V. Tarasov, No violation of the Leibniz rule. No fractional derivative. *Commun. Nonlin. Sci. Numer. Simul.* **18** (2013), 2945-2948; doi: 10.1016/j.cnsns.2013.04.001

– **COMFORMABLE FC**. R. Khalil (2014), T. Abdeljawad (2015), and many followers ;(

The so-called **comformable “fractional” derivatives T^α** are shown in several critical papers to be just differentiation of 1st order (for $0 < \alpha < 1$) with variable coefficients (depending on α) or of higher integer orders (for $\alpha > 1$). Thus all such publications treat problems for integer order equations, which is NOT observed (OR ignored) by the authors ...

$$T_\alpha f(t) := \lim_{\epsilon \rightarrow 0} \frac{f(t + \epsilon t^{1-\alpha}) - f(t)}{\epsilon}, \quad \text{but} \quad T_\alpha f(t) = t^{1-\alpha} f'(t).$$

A.A. Abdelhakim, J.A.T. Machado, A critical analysis of the comformable derivative. *Nonlin. Dynam.* **55**, 3063-3073 (2019); doi: 10.1007/s11071-018-04741-5

A.A. Abdelhakim, The flaw in the conformable calculus: It is conformable because it is *not* fractional. *Fract. Calc. Appl. Anal.* **22**, 242-254 (2019); doi: 10.1515/fca-2019-0016

D.R. Anderson, E. Camrud, D.J. Ulness, On the nature of the conformable derivative and its applications to physics. *J. Fract. Calc. Appl.* **10**, No 2, 92-135 (2019)

– **“FC” OPERATORS WITH REGULAR (NOT SINGULAR) KERNELS.** Say, C-F and A-B operators ...

Caputo-Fabrizio (C-F) derivative (say of Caputo type, similar one is or R-L type):

$${}^{CF}D_0^\alpha f(t) = \frac{M(\alpha)}{1-\alpha} \int_0^t \exp\left(-\frac{\alpha}{1-\alpha}(t-\tau)\right) f'(\tau) \tau,$$

where $M(\alpha)$ is a normalization factor such that $M(0) = M(1) = 1$.

Atangana-Baleanu (A-B) derivative, where the exp is replaced by a M-L function:

$${}^{ABC}D_0^\alpha f(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t E_\alpha\left(-\frac{\alpha}{1-\alpha}(t-\tau)^\alpha\right) f'(\tau) \tau,$$

$B(\alpha)$ has same role and properties as $M(\alpha)$. RL-type versions of the C-F and A-B operators have also appeared in the works.

The C-F derivative has an analytic kernel, while the A-B-C derivative has a kernel that is continuous but not differentiable at $t = 0$. At first sight, the fractional derivatives defined using non-singular kernels may appear very attractive since they avoid several difficulties that are caused by the singular nature of the RL and Dzhrbashyan-Caputo kernel. Thus, it is unsurprising that these simpler operators have become quite popular since their appearance some years ago.

But these operators with non-singular kernels have serious shortcomings that strongly discourage their use, and also should not be called to be of FC. In several critical papers, it was shown that such operators: - fail to satisfy the fundamental theorem of FC; - i.e. they do not allow the existence of a corresponding convolution integral (of fractional order) for which the derivative is the right-inverse. Even if the authors provide some definitions of “C-F and A-B integrals”, it is shown that the corresponding derivatives do not act as left-inverse, UNLESS the unnatural and restrictive condition $f(0) = 0$ is imposed on the function (so-called “zero-zero” property; the derivative at 0 is always 0).

Then, the C-F and A-B derivatives can be replaced by finite linear combinations of operators of that are already known from classical Calculus and Caputo derivative. That does not bring any news. [See for example:](#)

K. Diethelm, R. Garrappa, A. Giusti, M. Stynes, Why fractional derivatives with nonsingular kernels should not be used. *Fract. Calc. Appl. Anal.***23**, No 2, 610-634 (2020); doi: 10.1515/fca-2020-0032

M.D. Ortigueira, V. Martynyuk, M. Fedula, J.T. Machado, The failure of certain fractional calculus operators in two physical models. *Fract. Calc. Appl. Anal.***22**, No 2, 255-270 (2019); doi: 10.1515/fca-2019-0017

A. Hanuga, A comment on a controversial issue: A generalized fractional derivative cannot have a regular kernel. *Fract. Calc. Appl. Anal.***23**, No 1, 211-223 (2020); doi: 10.1515/fca-2020-0008

A. Giusti, A comment on some new definitions of fractional derivative. *Nonlyn. Dynam.* **93**, 1757-1763 (2018); doi: 10.1007/s11071-018-4289-8

We may continue with critical arguments for:

- The so-called Katugampola FC operators;
- The so-called ψ -analogues of FC operators;
- The k -analogues of operators,
that are based on the use of Γ_k -function, thus - NOT new ones;
- etc.

THANK YOU FOR ATTENTION !