Summer School Workshop "Modern Problems in PDEs and Applications"

11 August, 2023

Abstracts

Weyl-Hörmander calculus and applications to the analysis of PDEs

Julio Delgado (University of Valle, Colombia)

In this talk we present some applications of the Weyl-Hörmander $S(m, g)$ calculus to the analysis of PDEs, As applications we establish first the well-posedness for a class of degenerate Schrödinger equations with irregular potentials and secondly some spectral properties for a class of anharmonic oscillators by mean of the membership to corresponding Schatten-von Neumann ideals. Both results correspond to recent joint work with Duván Cardona, Marianna Chatzakou and Michael Ruzhansky.

Hardy and Rellich type identities related to Baouendi-Grushin operators

Nurgissa Yessirkegenov (Institute of Mathematics and Mathematical Modeling, Kazakhstan and Suleyman Demirel University, Kazakhstan)

In this talk, we discuss our recent progress on Hardy and Rellich type identities and inequalities related to Baouendi-Grushin operators. This talk is based on the joint research with Ari Laptev (Imperial College London, UK) and Michael Ruzhansky (Ghent University, Belgium), and with Amir Zhangirbayev (Suleyman Demirel University, Kazakhstan).

On the Benjamin-Ono equation in the half line

Duván Cardona (Ghent University, Belgium)

In [2] we consider the inhomogeneous Dirichlet initial–boundary value problem for the Benjamin-Ono equation formulated on the half line. We study the global-in-time existence of solutions to the initial–boundary value problem. Under a suitable weighted condition of the boundary term, we present an analogue of the result in Tao [5] about the well-posedness of the Benjamin-Ono equation on the real line in H^1. Our approach combines the techniques developed by Ionescu and

Kenig [3] and Ponce and Fonseca [4] with the Calderón commutator technique developed in [1].

This talk is based on my joint work with Liliana Esquivel.

References:

[1] Calderón A. P. Commutators of singular integral operators, Proc. Natl. Acad. Sci. USA, 53 (5) (1965), p. 1092

[2] Cardona, D., Esquivel, L. On the Benjamin-Ono equation in the half line., Nonlinear Analysis. Vol. 212, (2021), Art. 112427

[3] Ionescu A., Kenig C. Global well-posedness of the Benjamin-Ono equation in low-regularity spaces J. Amer. Math. Soc., 20 (3) (2007), pp. 753-798

[4] Fonseca G., Ponce G. The IVP for the Benjamin-Ono equation in weighted Sobolev spaces, J. Funct. Anal., 260 (2) (2011), pp. 436-459

[5] Tao T. Global well-posedness of the Benjamin–Ono equation in J. Hyperbolic Differ. Equ., 1 (01) (2004), pp. 27-49

Critical Sobolev type identities and inequalities

Yerkin Shaimerdenov (Suleyman Demirel University, Kazakhstan)

In this talk critical case of cylindrically extended Sobolev type (Improved Hardy) inequality will be discussed. Moreover, identities, higher order versions and some applications will be presented.

On the deficiency index of a differential operator with fast oscillating coefficients

Alibek Yeskermessuly (Altynsarin Arkalyk pedagogical Institute, Kazakhstan)

We study the deficiency indices of the minimal differential operator L_0 generated in $L_2(0,\infty)$ by the differential expression

$$ly = y^{(4)} - (h(x)y')' - q(x)y, \quad x \in (0;\infty),$$

where $q(x) \in C^{(2)}[0;+\infty)$ is a sufficiently smooth function satisfying the Titchmarsh-Levitan conditions, and $h(x)$ is a fast oscillating real function. Asymptotic formulas are constructed as $x \to \infty$ for the fundamental system of solutions of the equation $ly = \lambda y$. A model example is given.

Some new multidimensional Cochran-Lee and Hardy type inequalities

Markos Fisseha Yimer (Addis Ababa University, Ethiopia)

A multidimensional Cochran-Lee operator is introduced and investigated in the frame of Hardy-type inequalities with parameters $0 < p \leq q < \infty$. Moreover, for the case $p = q$ and power weights even the sharp constant is derived, thus generalizing the original Cochran-Lee inequality to a multidimensional setting. As applications both several known but also new inequalities are pointed out.

A Cauchy-Dirichlet problem for the Hadamard time-fractional diffusion equation

Asselya Smadiyeva (Institute Mathematics and Mathematical Modelling, Kazakhstan)

In this work we study the solvability of following Cauchy-Dirichlet problem

$$\begin{cases} D^{\alpha}_{a+,t}u(t,x) - (\log \frac{t}{a})^{\beta}\Delta_x u(t,x) = 0, \ \ t > a > 0, \ \ x \in \Omega, \\ \\ I^{1-\alpha}_{a+,t}u(a,x) = \phi(x), \ \ x \in \Omega, \\ \\ u(t,x) = 0, \ \ t > a, \ \ x \in \partial\Omega, \end{cases}$$

where $\beta > -\alpha$, Δ_x is N-dimensional Laplace operator, $D^{\alpha}_{a+,t}$, $I^{1-\alpha}_{a+,t}$ are the Hadamard fractional derivative, integral of order $\alpha \in (0,1)$, respectively.

Our approach is based on the classical Fourier method. In order to prove the existence of problem we use Fourier method and basic properties of the Kilbas-Saigo function $E_{\alpha,m,l}(z)$, which defined by [1, Remark 5.1]

$$E_{\alpha,m,l}(z) = \sum_{k=0}^{\infty} c_k z^k, \ \ c_0 = 1, \ \ c_k = \Pi_{j=0}^{k-1} \frac{\Gamma(\alpha(jm+l)+1)}{\Gamma(\alpha(jm+l+1)+1)}, \ \ k \geq 1,$$

where $m = 1, 2, 3, ...$, $l = -1, 0, 1, ...$, $\alpha \in (0,1)$.

References

[1] A. A. Kilbas, M. Saigo, On solution of integral equations of Abel-Volterra type. Differ. Integral Equ.– 8 – 15 – 1995, 993-1011.

Quantum groups

Yergen Aikyn (Ghent University, Belgium)

I will talk about quantum groups, give some examples of them and talk about the process of deformation quantization.