Pseudo-Differential Operators and Related Topics

PSORT-2024

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Pseudo-differential conference

BE HARMONIC WITH ANALYSIS

$$\int_{\hat{G}} \mathrm{Tr}(\pi(x)\sigma(x,\pi)\hat{f}(\pi))d\mu(\pi)$$

Make a difference: pseudo-differentiate

Integrate with mind but pseudo-differentiate with heart

Contents

Abstracts

Globally properties for a class of periodic-evolution operators on compact manifolds

Wagner Augusto Almeida de Moraes
Federal University of Paraná, Brazil

In this talk, I will present necessary and sufficient conditions for the global hypoellipticity and solvability for a class of periodic-evolution operators defined on compact manifolds. The method involves establishing conditions on the matrix symbol of the system. This allows for its transformation into a suitable triangular form, represented as a sum of a diagonal matrix and a nilpotent upper triangular matrix. Consequently, the study of global hypoellipticity focuses on analyzing the eigenvalues and their averages. This is a joint work with Fernando de Ávila Silva (UFPR-Brazil).

Restriction Theorems for Fourier-Dunkl transform

Pradeep Boggarapu
BITS Pilani, K K Birla Goa Campus, India

In this talk, we address the following Strichartz's restriction problem: For a given surface S embedded in $\mathbb{R}^d \times \mathbb{R}^n$ with $n+d \geq 2$, for what values of $1 \leq p < 2$, do we have

$$\left(\int_S |\widehat{f}(\xi,\zeta)|^2 h_\kappa^2(\xi) d\sigma(\xi,\zeta) \right)^{\frac{1}{2}} \leq C \|f\|_{L^p_\kappa(\mathbb{R}^d \times \mathbb{R}^n)}?$$

Here $\widehat{f}$ is the Fourier-Dunkl transform of f defined by

$$\widehat{f}(\xi,\zeta) = \frac{1}{c_\kappa (2\pi)^{n/2}} \int_{\mathbb{R}^d} \int_{\mathbb{R}^n} f(x,y) E_\kappa(-i\xi, x) e^{-iy\cdot\zeta} h_\kappa^2(x) dx dy, \tag{1}$$

for all $(\xi,\zeta) \in \mathbb{R}^d \times \mathbb{R}^n$ for suitable functions f on $\mathbb{R}^d \times \mathbb{R}^n$, where $E_\kappa(-i\xi, x)$ and $h_\kappa^2(x)$ are the Dunkl kernel and Dunkl weight function on $\mathbb{R}^d$ associated with a root system. In particular, we prove Strichartz's restriction theorem for the Fourier-Dunkl transform for certain surfaces, namely, cone, paraboloid, sphere, and hyperboloid, and its generalization to the family of orthonormal functions. Finally, as an application of these restriction theorems, we establish respected versions

of Strichartz estimates for wave propagator, Schrödinger's propagator and Klein-Gordon propagator associated with the Dunkl Laplacian. This restriction theorem generalizes the Stein-Tomas and Strichartz's restrictions theorems in the special cases.

This is a joint work with P Jitendra Kumar Senapati, Shyam Swarup Mondal and Hatem Mejjaoli.

The weak $(1,1)$ type of Fourier integral operators with complex phases.

Duván Cardona Sánchez

Ghent University, Belgium

A fundamental result due to Terence Tao establishes the weak $(1,1)$ type of Fourier integral operators of order $-(n-1)/2$ associated with real-valued phase functions. In this talk, we discuss the extension of Tao's estimate to the setting of Fourier integral operators of the same order and with complex phases. Joint work with Prof. Michael Ruzhansky.

Functional inequalities on Lie groups
(Based on joint works with A. Kassymov and M. Ruzhansky)

Marianna Chatzakou

Ghent University, Belgium

We will discuss a family of logarithmic inequalities on several Lie groups. The inequalities we will discuss are well-known in the Euclidean space and we will discuss their extension in the general setting of stratified groups and sometimes in the even more general setting of graded or even general Lie groups.

These inequalities include the log-Sobolev, Shannon, and Shannon inequality. Interestingly, restricting ourselves to the stratified setting, we also show the "semi-Gaussian" inequality which recovers the classical Gross inequality when the stratified group is simply $\mathbb{R}^n$. In the case of the Heisenberg group, the appearing constant allows us to pass to infinite dimensions, which in turn might give rise to an infinite-dimensional Heisenberg group with the first stratum of infinite dimension and a probability measure on it.

Convolutions on Lie groups, generic Bessel potential spaces and applications

Roland Duduchava

University of Georgia, Tbilisi State University, Georgia

The purpose of the presentation is to discuss the convolution integro-differential equations on Lie groups and their applications to some equations of Mathematical Physics. In this framework we suggest to underline the role of Generic Bessel potential spaces (GBPS) to the structure of underlying Lie group. Definition of GBPS are based on generic differential operators from the Lie algebra of the Lie group. Such generic Bessel potential spaces are adapted better to the investigation of integro-differential (of pseudo-differential) operators on Lie groups.

We concentrate investigation on a Lie groups $\{G, x\circ y\}$ with the group operation $x\circ y$, which are homeomorphic to the Lie group $\{\mathbb{R}^n, x\circ y = x+y\}$. Then on $\{G, x\circ y\}$ we have uniquelly defined Haar measure $d_G\mu$, the Fourier transform $\mathcal{F}_G$, its inverse $\mathcal{F}_G^{-1}$ and generic differential operators $\mathfrak{D}_1, \dots, \mathfrak{D}_n$, generated by the vector fields from the corresponding Lie algebra. The dual group is then $\widehat{G} = \mathbb{R}^n$ and Convolution operators are

$$\boldsymbol{W}^0_{a,G} := \mathcal{F}_G^{-1} a \mathcal{F}_G \ : \ \mathbb{S}(G) \to \mathbb{S}'(G), \tag{2}$$

where the symbol $a(\xi)$ is a distribution on the dual group $a \in \mathbb{S}'(\widehat{G}) = \mathbb{S}'(\widehat{\mathbb{R}^n})$, $\mathbb{S}(G)$ is the Schwartz spaces of fast decaying smooth functions and $\mathbb{S}'(G)$ is the spaces of distributions.

We will expose several examples of Lie groups and corresponding GBPS. Then we concentrate on the investigation of boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{C}$ with the Lipschitz boundary, containing a finite number of angular points (knots). The Dirichlet, Neumann and mixed type BVPs are considered in two different non-classical setting: A) Solutions are sought in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{C})$, $1 < p < \infty$, $s > 1/p$; A) Solutions are sought in the generic Bessel potential spaces with weight $\mathbb{GH}^s_p(\mathcal{C}, \rho)$. By the localization the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domains, also in cases of double angles. Explicit criteria for the Fredholm property and the unique solvability of the initial BVPs in both cases are obtained and, for the Generic Bessel potential spaces also singularities of solutions at knots of the mentioned BVPs are indicated explicitely.

The first part of the presentation is based on joint results with M. Ruzhanski, D, Cardona, A. Hendrix (Ghent) and the second part-on joint work with M. Caava, M. Tutberidze (Tbilisi).

Singular Sobolev spaces and applications to degenerate elliptic partial differential equations

Djamel Eddine Kebiche
University of Vienna, Austria

We deal with boundary-value problems of the form

$$\begin{cases} -\sum_{i,j=1}^{d} D_j(a_{ij}D_i u) + (b\cdot D)u + cu = f & \text{in } \Omega \\ \mathrm{Tr}(u) = 0 & \text{on } \partial\Omega \end{cases} \tag{3}$$

under the condition

$$\exists \mu, C > 0 \, \forall \xi \in \mathbb{R}^d : \, \mu w^4(x)|\xi|^2 \leq \sum_{i,j=1}^{d} a_{ij}(x)\xi_i\xi_j \leq C w^4(x)|\xi|^2 \text{ a.e. in } \Omega \tag{4}$$

where $w \in \mathcal{C}^1(\Omega)$ and $\{x \in \Omega \mid w(x) = 0\}$ is a Lebesgue negligible set. We highlight that no assumption on the integrability of w^{-1} is considered, i.e. the function w can be arbitrary small on Ω. In this setting, the theory of weighted Sobolev spaces fails to solve the boundary-value problem (3) since the function w may not satisfy condition $B_2(\Omega)$, and hence, the related weighted Sobolev space may not be Cauchy complete. To deal with the boundary-value problem (3), the author developed in [1] the notion of *singular Sobolev space,* which is naturally associated with an *elimination function* w and with a new notion of weak derivative, called *the weak derivative in the sense of* $L^1_{w,\mathrm{loc}}(\Omega)$. An elimination function w on $\Omega \subseteq \mathbb{R}^d$ is a measurable function which is non-null and finite almost everywhere. The singular Sobolev space $L^p_w(\Omega)$ is defined as the space of measurable functions u such that $wu \in L^p(\Omega)$. Since the elimination function can be arbitrary small, elements of $L^p_w(\Omega)$ are not necessary locally integrable, and hence the distributional derivative, which is not well defined in this setting, is replaced by the notion of weak derivative in the sense of $L^1_{w,\mathrm{loc}}(\Omega)$. The latter is defined using a suitable integration by part formula.

For a given family $\{w_\alpha\}$ ($\alpha \in \mathbb{N}$ with $|\alpha| \leq m$, $m \in \mathbb{N}$) of elimination functions, the space $W^{m,p}_{\{w_\alpha\}}(\Omega)$ is defined as the subspace of $L^p_w(\Omega)$ ($w := w_0$) of functions whose α-weak derivative in the sense of $L^1_{w,\mathrm{loc}}(\Omega)$ exists and belongs to $L^p_{w_\alpha}(\Omega)$ for all α. A condition on $\{w_\alpha\}$ under which the space $W^{m,p}_{\{w_\alpha\}}(\Omega)$ is Cauchy complete is given (Thm. 14 of [1]).

In case the domain Ω has a sufficiently smooth boundary, a trace operator Tr on $W^{1,p}_{\{w^{|\alpha|+1}\}}(\Omega)$ (with values in $L^p(\partial\Omega)$) can be defined provided $|w| > \delta > 0$ on $\Omega\backslash K$ where $K \Subset \Omega$, or $\nabla w \in L^\infty(\Omega)$ (see Sec. 2.4 of [1]). Several results such as Poincaré inequality (adapted to our framework) and density of smooth functions are proved.

The existence of a unique weak solution in $W^{1,2}_{\{w^{|\alpha|+1}\},0}(\Omega)$ to the boundary-value problem (3) is proved. Moreover, assumptions under which the solution is not locally integrable are given, and an example of a boundary-value problem with a non-locally integrable solution is given as well. This shows in particular that the theory of degenerate Sobolev spaces introduced by Sawyer and Wheeden in [3] is not well fitted to solve the boundary-value problem (3) (under condition (4) with w

arbitrary small) since elements of a degenerate Sobolev space are necessary locally integrable.

References

[1] Kebiche. D., Singular Sobolev spaces and applications to degenerate elliptic partial differential equations. Submitted to The Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, available at `https://arxiv.org/abs/2311.12635`

[2] Kebiche. D., Density of smooth functions in singular Sobolev spaces and applications to degenerate elliptic partial differential equations. Finished preprint. Planned to be submitted in The Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, available at `https://drive.google.com/file/d/10on46RRQl59_-hdqa4Xv5drZs-23rAS6/view?usp=sharing`

[3] Sawyer, E. T., & Wheeden, R. L. (2010). Degenerate Sobolev spaces and regularity of subelliptic equations. Transactions of the American Mathematical Society, 362(4), 1869–1906. http://www.jstor.org/stable/40590893

[4] Cavalheiro, Albo. (2008). Weighted Sobolev Spaces and Degenerate Elliptic Equations. Boletim da Sociedade Paranaense de Matemï¿½tica. 26. 10.5269/bspm.v26i1-2.7415.

Global pseudodifferential operators on graded Lie groups

Eske Ewert

University of Hannover, Germany

The Lie algebra of a graded Lie group G is equipped with a family of anisotropic scalings. This allows to redefine the order of left-invariant differential operators and polynomials on G. In this talk, I will explain how this leads to a pseudodifferential calculus on G which is analogous to the global calculus defined by Shubin and Helffer on $\mathbb{R}^n$. To define the calculus, we follow the approach of van Erp and Yuncken using an appropriate tangent groupoid. ¿ The operators have a principal symbol in a noncommutative algebra. Ellipticity is then checked using the unitary representations of G, similar to the Rockland condition. In this case, the operator is Fredholm on certain adapted Sobolev spaces. While in the classical case its index can be computed by an integral over $T^*\mathbb{R}^n$, we obtain an integral formula on $G \times \widehat{G}$.

This talk is based on joint work with Philipp Schmitt and Ryszard Nest.

Weyl calculus on graded Lie groups

Serena Federico
University of Bologna, Italy

In this talk, we will present a family of pseudo-differential calculi on graded nilpotent Lie groups, into which we will identify the Weyl quantization in this setting. We will first introduce the family of quantizations called τ-quantizations, and establish the corresponding pseudo-differential τ-calculi. Afterwards, by restricting ourselves to the subfamily of symmetry τ-quantizations, we will identify the Weyl quantization in the Heisenberg group by following specific rigorous criteria. Through similar considerations, we will finally justify why the same quantization is the best candidate Weyl quantization on any graded nilpotent Lie group. This talk is based on a joint work with D. Rottensteiner and M. Ruzhansky.

Quantum Limits on step-2 Nilmanifolds

Steven Flynn
University of Padova, Italy

We develop a semi-classical symbol calculus for 2-step nilmanifolds and use it to determine invariance properties of quantum limits for the sub-Laplacian.

Weyl laws and closed geodesics on typical manifolds

Jeffrey Galkowski
University College London, UK

We discuss the typical behavior of two important quantities on compact Riemannian manifolds: the number of primitive closed geodesics of a certain length and the error in the Weyl law. For Baire generic metrics, the qualitative behavior of both of these quantities has been well understood since the 1970's and 1980's. Nevertheless, their quantitative behavior for typical manifolds has remained mysterious. In fact, only recently, Contreras proved an exponential lower bound for the number of closed geodesics on a Baire generic manifold. Until now, this was the only quantitative estimate on the number of geodesics for typical metrics, and no such estimate existed for the remainder in the Weyl law. In this talk, we give stretched exponential upper bounds on the number of primitive closed geodesics for typical metrics. Furthermore, using recent results on the remainder in the Weyl law, we will use our dynamical estimates to show that logarithmic improvements in the remainder in the Weyl law hold for typical manifolds. The notion of typicality used in this talk is a new analog of full Lebesgue measure in infinite dimensions called predominance. Based on joint work with Y. Canzani.

L^p-estimates in Safarov pseudodifferential calculus on manifolds

David Santiago Gómez Cobos
Ghent University, Belgium

Safarov introduced a pseudodifferential calculus on smooth manifolds without using the local theory of $\mathbb{R}^n$, instead he used a linear connection ∇ to obtain a global calculus. Using this construction and assuming that the connection is symmetric he was able to weaken the classical restriction $\rho > 1/2$ to $\rho > 1/3$. We will introduce the Safarov pseudodifferential calculus and we will present a result on $L^p - L^p$ estimates for this calculus.

On Pleijel's theorem for Dirichlet realization of SubLaplacians

Bernard Helffer
Nantes Université, France

We will discuss extensions of Courant's theorem and Pleijel's theorem to the case of SubLaplacians. Examples are treated in the case of rank 2 nilpotent groups and sub-riemannian manifolds. This is a work in progress with Rupert Frank and Cyril Letrouit.

Fourier analysis on lattices and beyond

Arne Hendrickx
Ghent University Belgium

We will first discuss how we can construct a Fourier analysis on fundamental domains of lattices in $\mathbb{R}^d$. Once this is in place, we consider L^p-L^q boundedness of Fourier multipliers. Furthermore, we investigate Titchmarsh theorems in this setting. The first Titchmarsh theorem ensures L^p-integrability of the Fourier transform of a function for a range of p under a certain Lipschitz-condition. The second one characterises the space of functions satisfying this Lipschitz-condition in terms of the decay of the Fourier transform of those functions. Lastly, we will briefly discuss pseudo-differential operators on fundamental domains of lattices and on lattices, as well as perspectives for crystals and quasi-crystals.

Maximal Sobolev Regularity

Christian Jäh
Georg-August-Universität Göttingen

In this talk, we present ongoing work surrounding maximal Sobolev regularity at radial points. This work continues the classical study of Guillemin and Schaeffer in [1] which we will introduce and then discuss the progress in handling normal forms. The normal form of the operator is of second order and the main task is to construct an appropriate parametrix. We will discuss precise regularity results for solutions to pseudodifferential equations with real principal symbol near isolated radial points, microlocally in the scale of Sobolev spaces H^s. Compared to operators of real principal type, the new phenomenon of *maximal Sobolev regularity* appears and the index of maximal Sobolev regularity can be effectively computed in terms of the sub-principal symbol of the operator under consideration.

This is joint work with Ingo Witt continuing/fleshing out the arguments and results of [2]

[1] Guillemin, Victor and Schaeffer, David, *On a certain class of Fuchsian partial differential equations*, Duke Math. J., 44,1 (1977), pp. 157–199.

[2] Witt, Ingo, *Maximal Sobolev regularity at radial points*, Symmetries in algebra and number theory (SANT), Göttingen Universitätsverlag, 2009, 978-3-940344-96-0, pp. 149–160.

Pseudodifferential operators on Banach function spaces

Oleksiy Karlovych
Universidade Nova de Lisboa, Portugal

We prove a boundedness result for pseudodifferential operators on Banach function spaces. We pay a special attention to the case of variable Lebesgue spaces with exponents beyond the class of log-Hölder continuous exponents. Further we present a sufficient condition for the Fredholmness of pseudodifferential operators with slowly oscillating symbols on variable Lebesgue spaces. Both theorems generalize pioneering results by Rabinovich and Samko for variable Lebesgue spaces with log-Hölder continuous exponents.

Global Properties for Systems of Vector Fields on Compact Lie Groups

Alexandre Kirilov
Federal University of Paraná, Brazil

In this talk, I will share our initial findings on hypoellipticity and global solvability of overdetermined systems of vector fields defined on compact Lie groups. These results extend well-known findings in the torus and introduce new examples not yet explored in the literature.

Phase space Feynman path integrals of parabolic type on the torus as analysis on path space

Naoto Kumano-go
Kogakuin University, Japan

We provide general sets of functionals for which parabolic phase space Feynman path integrals on the torus have a mathematically rigorous meaning. More exactly, for each functional belonging to each set, the time slicing approximation of the phase space path integral converges uniformly on compact subsets to some function of the ending point of position paths and the starting point of momentum paths. Each set of functionals is closed under addition, multiplication, translation, invertible integer linear transformation, and functional differentiation. As a result, we can create a large number of path integrable functionals.

Though we must exercise caution when using phase space path integrals, several properties comparable to those of conventional integrals are applicable.

Quantum time-frequency analysis and pseudodifferential operators

Franz Luef
NTNU, Norway

We propose an extension of quantum harmonic analysis which is based on the notion of the translation of an operator by completing it with a suitable modulation of an operator, the associated time-frequency shift of an operator is the key player in our theory of quantum time-frequency analysis. We discuss relations between this framework and time-frequency representations and Weyl quantization as well as boundedness results of pseudodifferential operators on modulation spaces.

This is based on joint work with Henry McNulty (Cognite AS, Oslo) and Monika Doerfler (Univ. Vienna).

Asymptotic Properties for the Spectrum of Certain Semiregular Global Systems

Marcello Malagutti

University College London, UK

In this talk I present some recent results about spectral analysis of systems of PDEs related also with quantum physics models. Namely, it is stated a Weyl asymptotic for a class of systems containing certain quantum optics models such as the Jaynes-Cummings model and the Raby one, but also models of geometric differential complexes over $\mathbb{R}^n$. In more detail, it is provided the asymptotics of the Weyl spectral counting functions in terms of the principal, semiprincipal and subprincipal symbols of the system. Moreover, a quasi-clustering result for this class of positive systems is given. In fact, we show the eigenvalues of this kind of systems can be located in the union of intervals having centers determined by the matrix invariants of the semiprincipal and subprincipal symbol and width decreasing as the centers go to $+\infty$. Finally, a meromorphic continuation of the spectral zeta function for semiregular Non-Commutative Harmonic Oscillators (NCHO) is given. By "semiregular system" we mean a pseudodifferential systems with a step $-j$ in the homogeneity of the jth-term in the asymptotic expansion of the symbol.
The aforementioned results were obtained jointly with Alberto Parmeggiani.

Index Whittaker transform based on pseudo-differential operators

Jeetendrasingh Maan

National Institute of Technology Hamirpur, India

Convolution inequality in a family of weighted Lebesgue spaces by using the theory of index Whittaker transform are argued. An iterated convolution is defined and obtained its estimate. Test function spaces are introduced and discussed the continuity of the index Whittaker transform, differential operator on these spaces. Moreover, pseudo-differential operator (p.d.o) associated with index Whittaker transform is defined and studied its continuity between the given function spaces. Lastly, some applications of the index Whittaker transform in pseudo-differential equation are discussed.

Multilinear Fourier Integral operators on modulation space

Lalit Mohan

Indian Institute of Technology Delhi, India

In this article, we study properties of multilinear Fourier integral operators on weighted modulation spaces. In particular, using the theory of Gabor frames, we study boundedness of multilinear Fourier integral operators on products of weighted modulation spaces. Further, we investigate the periodic multilinear Fourier integral operator. Finally, we study continuity of bilinear pseudo-differential operators on modulation spaces for certain symbol classes, namely **SG**-class.

In this talk, I will discuss results of my following paper:

- A. Dasgupta, L. Mohan and S.S. Mondal, "Multilinear Fourier Integral operators on modulation space", Forum Math., accepted (2023).

Sharp maximal function estimates for multilinear pseudo-differential operators

Bae Jun Park

Sungkyunkwan University, South Korea

In this talk, we will study pointwise estimates for linear and multilinear pseudo-differential operators with exotic symbols in terms of the Fefferman-Stein sharp maximal function and Hardy-Littlewood type maximal function. Especially in the multilinear case, we use a multi-sublinear variant of the classical Hardy-Littlewood maximal function introduced by Lerner, Ombrosi, Pérez, Torres, and Trujilo-González, which provides more elaborate and natural weighted estimates in the multilinear setting. This is based on joint work with Naohito Tomita.

L^p-boundedness of pseudo-differential operates on radial sections of line bundles over the Poincaré upper half plane

Tapendu Rana

Ghent University, Belgium

For a given function $a(x,\xi)$ on $\mathbb{R}^n \times \mathbb{R}^n$, consider the pseudo differential operator $a(x,D)$ defined by

$$a(x,D)(f) = \int_{\mathbb{R}^n} a(x,\xi)\widehat{f}(\xi)e^{2\pi ix\cdot\xi}d\xi,$$

where $\widehat{f}$ is the Fourier transform of a function f. Let S^0 be the set of all smooth functions $a : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{C}$ satisfies,

$$\left|\partial_x^\beta \partial_\xi^\alpha a(x,\xi)\right| \leq C_{\alpha,\beta}\,(1+|\xi|)^{-|\alpha|}$$

for all $x, \xi \in \mathbb{R}^n$ and for all multi indices α and β. Then the following result is well known:

Theorem. *For $a \in S^0$, $a(x, D)$ extends to a bounded operator on $L^p(\mathbb{R}^n)$ to itself, for $1 < p < \infty$.*

In this talk, we will discuss an analogue of this result on radial sections of line bundles over the Poincaré upper half plane. Additionally, we will explore the case where the symbol exhibits restricted regularity in the spatial variable.

This is a joint work with Prof. Michael Ruzhansky.

Characterization of wave front sets associated with ultradistributional spaces by wavelet transform

Ashish Pathak

Banaras Hindu University, Varanasi, India

In the current study, we identified the special wavelet transform given by Moritoh in ultradistributional spaces and using this special wavelet transform we defined and characterized wave front sets for ultradistributions. We also discuss some examples and identify wave front set corresponding to it.

Regularity of partial integro-differential equations on locally compact groups

Joel Esteban Restrepo Tangarife

Ghent University, Belgium

We discuss existence, uniqueness, norm estimates and other properties of solutions of partial integro-differential equations on a locally compact group. Illustrative examples are also given.

Recent results on the norm of localization operators

Federico Riccardi

Politecnico di Torino, Italy

Time-frequency localisation operators (with Gaussian windows) were introduced by Daubechies in 1988. Since then they have been intensively studied, especially in terms of boundedness, compactness, shadow properties and estimates for the eigenvalues. However, there are few sharp estimates for the norm of these operators. In this talk, we will present a classical result of Lieb and two new results that provide sharp estimates for the operator norm of localisation operators under the assumption that the weight function belongs to one or more L^p-spaces. Moreover, we will also present a stability result for the Hilbert-Schmidt norm of localisation operators with a characteristic function as weight.

$L^p_\alpha(\mathbb{R}^{n+1}_+)$– Boundedness of Pseudo-Differential Operators involving the Weinstein Transform

Mohd Sartaj

Indian Institute of Technology (BHU) Varanasi, India

In this presentation, an $L^p_\alpha(\mathbb{R}^{n+1}_+)$ - boundedness of pseudo differential operators associated with class of symbol S^0 are proven by utilizing the theory of the Weinstein transform. Using the aforesaid theory various properties and boundedness results on $L^p_\alpha(\mathbb{R}^{n+1}_+)$ - type Sobolev spaces are given.

Regularity of oscillatory integral operators arising in evolutionary PDEs

Wolfgang Staubauch

Uppsala University, Sweden

We give a survey of oscillatory integral operators that arise naturally in connection to evolutionary partial differential equations, such as the Wave and Schrödinger equations. We also survey some recent regularity results for these oscillatory integral operators, with the emphasis on those which imply continuous dependence on the initial data, for fixed time, for evolutionary PDEs.

Bounded approximate identity for quasi-Banach spaces with applications to factorizations of Wiener-Lebesgue spaces and pseudo-differential operators

Joachim Toft

Linnaeus University, Sweden

Young's inequality ensures that $L^1 * L^p \subseteq L^p$ when $p \in [1, \infty]$. Around 65 years ago, it was proved by Cohen, Hewit and Rudin (in various contributions) that indeed $L^1 * L^p = L^p$ if we avoid $p = \infty$. That is, when $p \in [1, \infty)$. We observe that the identity is non-trivial, because L^1 does not contain any identity element. In the most general setting, the latter equality follows from the fact that L^1 possess the property on so-called bounded approximation identity. In the talk we explain that similar properties hold true when L^1 is replaced by any quasi-Banach algebra and and the product $*$ is replaced by suitable multiplications with quasi-Banach modules. As applications we show among others that $L^1 * L^p = L^p$ for $p \in [1, \infty)$ can be improved into $WL^{1,r} * L^p = L^p$, $p \in [1, \infty)$, $r \in (0, 1]$. Here $WL^{1,r}$ is a Wiener Lebesgue space which strictly increases with r and such that $WL^{1,1} = L^1$. We also give some links on further applications, e.g. concerning spans of Toeplitz operators. We also show that if s_p denotes the set of all symbols in S' such that corresponding pseudo-differential operator is a Schatten operator of order $p \in (0, \infty)$, then $WL^{1,r} * s_p = s_p$ If in addition $r \leq p$.

The talk is based on collaborations with Divyand Bhimani.

Phase space analysis of Hermite-type semigroups

S. Ivan Trapasso
Politecnico di Torino, Italy

We study the Hermite operator $H = -\Delta + |x|^2$ in $\mathbb{R}^d$ and its fractional powers H^β, $\beta > 0$ in phase space. Precisely, we consider phase-space representations of functions such as the short-time Fourier (alias Fourier-Wigner or Bargmann) transform and we measure regularity and decay in phase space by means of mixed Lebesgue norms - that is, resorting to modulation spaces $M^{p,q}$, $0 < p, q \leq \infty$. Applications to well-posedness results for (nonlinear) heat equations associated with the harmonic oscillator and its fractional powers will be discussed as well. Recent results in the same spirit will be addressed also for the twisted Laplacian (also known as the special Hermite operator) and its spectral multipliers. In spite of the close connection to the Hermite semigroup, techniques of pseudo-differential calculus are inhibited due to the lack of global ellipticity in this setting, therefore a phase space approach must rely on different pathways.

Fourier multipliers on quantum torus and their applications

Kanat Tulenov
Ghent University, Belgium

In this work, we discuss Fourier multipliers on quantum torus and their L^p -L^q boundedness, as well as their complete boundedness properties. We also present some applications.

Symmetrically Global Pseudo-Differential Operators involving the Weinstein Transform

Santosh Kumar Upadhyay
Indian Institute of Technology (BHU) Varanasi, India

In this talk, boundedness and compactness results for symmetrically global pseudo-differential operator on $L^p_\alpha(\mathbb{R}^{n+1}_+)$- type Sobolev space $\mathcal{H}^{r,s,p}_\alpha$ of order (r, s) are investigated by exploiting the theory of the Weinstein transform. Using symmetrically global symbol $\sigma(x, \xi)$ in S^{m_1,m_2}, $m_1, m_2 \in \mathbb{R}$, we have discussed various properties of minimal-maximal pseudo-differential operators involving the Weinstein transform. The weak solution of the symmetrically global pseudo-differential equation is obtained by using the aforesaid theory.

Weyl transforms associated with Dunkl wavelet transform

Randhir Verma
Indian Institute of Technology (ISM), Dhanbad, India

We define the Weyl transform associated with Dunkl wavelet transform and discuss its boundedness and compactness on the Lebesgue spaces.

Microlocal analysis for very weak solutions

Hans Vernaeve
Ghent University, Belgium

Many PDE, including the wave equation as a fundamental example, fail to have weak (i.e., distributional) solutions if the coefficients are not regular enough. As a remedy, the concept of a very weak solution was developed, and consists of a particular net $(u_\varepsilon)_\varepsilon$ of smooth functions solving the equation in an approximate sense. Since u_ε are smooth functions, one cannot read singular behaviour from each u_ε separately, but only through the growth behaviour for $\varepsilon \to 0$. In particular, one would like to understand propagation of singularities in this setting. We review some results about wave front sets of very weak solutions. The results build on the work of many authors in Colombeau algebras.

On elliptic problems with changing boundary conditions

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We study boundary problems for elliptic differential and pseudodifferential operators, where the boundary is divided into two parts and different elliptic boundary conditions are imposed along the two parts of the boundary. An archetypal example is the Zaremba problem for the Laplace operator in a bounded domain in $\mathbb{R}^n$ with the Dirichlet and the Neumann boundary condition imposed in different parts of the boundary. We approach this problem following a strategy proposed by B.-W. Schulze using a boundary reduction. This boundary reduction leads to an edge-degenerate boundary problem in the boundary which in turn is discussed in the light of a calculus for edge-degenerate boundary problems recently proposed by X.-C. Liu, Z.-P. Ruan, and myself.

Generalized Sobolev type spaces involving the Weinstein transform

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In this paper, the space $G_{\omega}^{p,s}\left(\mathbb{R}_{+}^{n+1}\right)$ is considered, and many properties, including completeness and inclusion, are discussed by using the theory of the Weinstein transform. It is shown that the space $S_{\omega}\left(\mathbb{R}_{+}^{n+1}\right)$ is dense in the space $G_{\omega}^{p,s}\left(\mathbb{R}_{+}^{n+1}\right)$. The generalized Hankel potential H^{k} associated with the Weinstein transform is introduced, and its properties are examined. The space $W_{\omega}^{m,p}\left(\mathbb{R}_{+}^{n+1}\right)$ is defined, and it is shown that the generalized Hankel potential H^{t} is an isometry of $W_{\omega}^{m,p}\left(\mathbb{R}_{+}^{n+1}\right)$ onto $W_{\omega}^{m+t,p}\left(\mathbb{R}_{+}^{n+1}\right)$.

Author Index