Direct and Inverse Problems with Applications, and Related Topics: A Comprehensive Summer School in Mathematical Analysis

August 19-23, 2024, Ghent, Belgium

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Programme

Organising committee:

- Prof Michael Ruzhansky (Ghent Analysis & PDE Center, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium)
- Dr Karel Van Bockstal (ibidem)
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General information

Location: The summer school takes place in a hybrid form, with the in-presence events being held at the Lecture room 3.1, Building S8, Campus Sterre.

Zoom link: `https://us02web.zoom.us/j/83095584409?pwd=c3huaUhZcWFrZ2NNMU05Ujg4aUlWQT09`

Internet access: Participants can either login using personal EDUROAM access at your home university, or select network "UGentGuest" and enter the credentials

login: guestSummer
passcode: JR7c7L2n

Timetable

Time	Monday August 19	Tuesday August 20	Wednesday August 21	Thursday August 22	Friday August 23
9:00-9:30	R/O	R	R	R	R
9:45-10:30	Luis Vega	Luis Vega	Jean Dolbeault	Cristiana Sebu	Bianca Stroffolini
Coffee break					
11:00-11:45	Luis Vega	Cristiana Sebu	Jean Dolbeault	Hajer Bahouri	Baoxiang Wang
11:45-12:30	Alessia Kogoj	Cristiana Sebu	Jean Dolbeault	Hajer Bahouri	Baoxiang Wang
Lunch break					
14:00-14:45	Wolfram Bauer	Wolfram Bauer	Bianca Stroffolini	Michael Anoussis	Mansur I. Ismailov
14:45-15:30	Wolfram Bauer	Alessia Kogoj	Michael Anoussis	Michael Anoussis	Mansur I. Ismailov
Coffee break					
16:00-16:45	Semyon Yakubovich	OE	Semyon Yakubovich	Bianca Stroffolini	Baoxiang Wang
16:45-18:00	Welcome cake	OE	PS + D	PS + D	PS + D + C
18:00-19:00	PS + D	Pizza	Dinner for speakers		Farewell samosa's

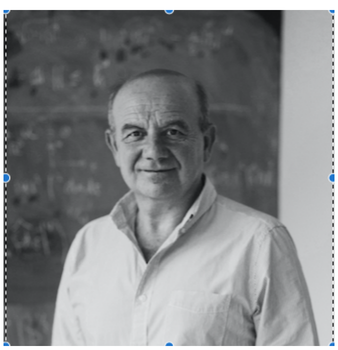
R:= Registration; O:=Opening; C:=Closing
L:= Lecture;
D:= Discussion;

PS:= Problem Sessions
PP:= Participants presentations (posters)
OE:= Outreach Event = Public Lecture + D + PP

The public lecture entitled

"Mathematics and Turbulence"

will be given by Prof Luis Vega on Tuesday, August 20, at 16:00.



Abstracts

Michael Anoussis (University of the Aegean, Greece, mano@aegean.gr)

Fourier algebras and homomorhisms

The Fourier algebra $A(G)$ and the Fourier-Stieltjes algebra $B(G)$ of a locally compact group G were introduced by Eymard in [4]. If G is abelian, $A(G)$ and $B(G)$ can be identified, via the Fourier transform, with $L^1(\widehat{G})$ and the measure algebra $M(\widehat{G})$ of the dual group $\widehat{G}$, respectively. Cohen characterized the homomorphisms from $A(H)$ into $B(G)$ for H and G locally compact abelian groups using a characterization of idempotents in $B(G)$ [2]. Homomorphisms of Fourier algebras for general locally compact groups were studied by Ilie-Spronk [5] and Daws [3].

In these talks, we will present basic properties of Fourier and Fourier-Stieltjes of locally compact groups and will discuss the results of Cohen and Ilie-Spronk and Daws. In the third talk we will show how the the existence of idempotents of arbitrarily large norm in the Fourier-Stieltjes algebra $B(G)$ is related to the existence of homomorphisms of arbitrarily large norm of Fourier and Fourier-Stieltjes algebras [1].

Hajer Bahouri (Sorbonne Université, France, hajer.bahouri@sorbonne-universite.fr)

Dispersion phenomena with applications to nonlinear evolution equations

The aim of these lectures is to give an overview about dispersion phenomena on the Euclidean space $\mathbb{R}^d$ as well as on some non commutative Lie groups. Then, to apply these properties to investigate some nonlinear evolution equations.

Wolfram Bauer (Leibniz Universität, Hannover, Germany, bauer@math.uni-hannover.de)

Subriemannian Geometry and Analysis of Hypoelliptic PDE

Subriemannian (SR) manifolds model motions under non-holonomic constraints and vastly generalizes the theory of Riemannian geometry. During the last years global and local aspects of such geometries, as well as their interactions, have attracted growing attention. In particular, - generalizing the Laplace-Beltrami operator in Riemannian geometry - one considers the hypoelliptic sublaplacian on M. The theory of hypoelliptic operators has been developed since the 70th and in the above framework of SR geometry it has seen some revival during recent years. In fact, SR structures and induced hypoelliptic PDEs nowadays arise in many different areas of mathematics, physics or even applied sciences including control theory, hypoelliptic diffusion, geometric measure theory, potential analysis, optimal transport or harmonic analysis.

During three introductory lectures we will discuss some aspects of the underlying geometry and analysis. Basic notions in SR geometry, examples of SR manifolds and applications will be presented. We take a closer look at the hypoelliptic sublaplacian and the induced heat equation which is an interesting (and standard) tool for considering inverse spectral problems in SR geometry. If time permits we discuss the existence of fundamental solutions for homogeneous Hörmander operators via lifting theorems by G.B. Folland and E.M. Stein. The latter results are based on work by S. Biagi and A. Bonfiglioli.

Jean Dolbeault (Université Paris-Dauphine, France, dolbeaul@ceremade.dauphine.fr)

Stability in functional inequalities

Obtaining explicit stability estimates in classical functional inequalities like the Sobolev inequality has been an essentially open question for 30 years, after the celebrated but non-constructive result [6] of G. Bianchi and H. Egnell in 1991. Recently, new methods have emerged which provide some clues on these fascinating questions. The goal of the course is to give an introduction on stability in some fundamental functional inequalities and present several methods that can be used to obtain explicit estimates.

Topics:

1. The Sobolev inequality and the non-constructive stability result of Bianchi–Egnell using concentration-compactness methods [6, 18]
2. Duality and stability in Hardy-Littlewood-Sobolev inequalities [12]
3. Entropy methods on the Euclidean space [19, 8]
4. Stability results for Gagliardo-Nirenberg inequalities on the Euclidean space [7, 8]
5. Stability results on the sphere and on the Gaussian space seen as an infinite dimensional limit of spheres [17, 13, 16, 11, 9, 10]
6. A constructive stability result for the Sobolev and the logarithmic Sobolev inequalities [14, 15]

Alessia Kogoj (Università di Urbino Carlo Bo, Italy, alessia.kogoj@uniurb.it)

An inverse mean value property for evolution equations

Aharonov, Shiffer and Zalcman, in 1981, proved the following rigidity theorem. Let D be a solid open subset of $\mathbb{R}^n$ and let z be a point of D. Assume that the Newtonian potential of D is proportional, outside D, to the potential of a mass concentrated at z. Then D is a Euclidean ball centered ad z. Suzuki and Watson, in 2001, extended this rigidity result to the heat balls, i.e., to the super level set of the fundamental solution of the heat equation. In our lecture, we show how these results can be extended to a class of hypoelliptic evolution Partial Differential Equations of Hörmander-type.

Mansur I. Ismailov (Gebze Technical University, Turkey, mismailov@gtu.edu.tr)

Direct and Inverse Scattering Problems for First Order Hyperbolic Systems with Time-dependent Potential

The non-stationary scattering problems for the first-order hyperbolic system of $n > 2$ equations on the whole axis and on the semi-axis are investigated, considering k (where $1 < k < n$) incident waves and $n - k$ scattered waves. Various problems with the same system but different boundary conditions are explored. The inverse scattering problems (ISPs) are studied on the whole axis, as referenced in [20, 21, 22], and on the semi-axis for various cases of incident and scattered waves are examined, as discussed in references [23, 24], followed by the analysis of matrix scattering operators on both the whole axis and the semi-axis. The uniqueness of the investigated ISPs are demonstrated by reducing their to Gelfand-Levitan-Marchenko type integral equations. Examples on the semi-axis illustrate that (a) a single scattering problem is insufficient for unambiguous reconstruction of the potential, and (b) the condition on the transmission matrix in boundary conditions is crucial, as referenced in [25, 26]. Subsequently, the ISPs are employed for the integration of nonlinear systems of N-waves with two velocities in $(2 + 1)$ dimensions using the inverse scattering transform method, as detailed in [27, 28, 29, 30].

Cristiana Sebu (University of Malta, Malta, cristiana.sebu@um.edu.mt)

Introduction to inverse problems and electrical impedance tomography

Lectures 1 and 2: Inverse problems arise in practical applications whenever there is a need to interpret indirect measurements. These lectures offer a brief introduction to linear and non-linear ill-posed inverse problems arising in practice and explain how to identify their ill-posedness. The guiding examples are the backward heat conduction problem and the inverse conductivity problem (known as Electrical Impedance Tomography).

Lecture 3: The talk is focused on recent developments of reconstruction algorithms that can be used to approximate conductivity distributions in Electrical Impedance Tomography from boundary measurements of input currents and induced potentials. The algorithms are non-iterative and are based on integral equation formulations.

Bianca Stroffolini (University of Naples, Italy, bstroffo@unina.it)

Asymptotic mean value formulas for the p-Laplacian in the Euclidean space and the Heisenberg Group

The classical mean value property characterizes harmonic functions. It can be extended to characterize solutions of many linear equations. We will focus in an asymptotic form of the mean value property that characterizes solutions of nonlinear equations. This question has been partially motivated by the connection between Random Tug-of-War games and the normalized p-Laplacian equation discovered some years ago, where a nonlinear asymptotic mean value property for solutions of a PDE is related to a dynamic programming principle for an appropriate stochastic game.

I will recall some basic mean value formulas for the Laplacian. I will talk about tug-of-war mean values formulas in the Euclidean setting . The mean value formulas for the p-Laplacian will be defined accordingly. This definitions require the knowledge of viscosity sub solution and super solution and also of the infinite-Laplacian.

Next, I'll give a PDE interpretation of general asymptotic mean value formulas based on a dynamic programming principle in the Euclidean setting.

The last lesson will be devoted to the Heisenberg Group setting.

Luis Vega (University of the Basque Country UPV/EHU, Spain, luis.vega@ehu.eus)

The binormal flow: a toy model for turbulence

I'll give an introduction to the use of the binormal curvature flow to describe the evolution of a vortex filament. This geometric flow is directly connected to the focusing 1d cubic non-linear Schrödinger equation whose Cauchy problem will be also addressed.

Baoxiang Wang (Jimei University, China, wbx@jmu.edu.cn)

Very rough function spaces and their applications to nonlinear evolution equations

We will introduce a new class of very rough function spaces, then apply them to study Navier Stokes, nonlocal NLS, NLKG equations and obtain some global existence and uniqueness of solutions for the above equations. A class of large initial data including distribution data can be handled in our results.

Semyon Yakubovich (University of Porto, Portugal, syakubov@fc.up.pt)

Some trends in the theory of integral transforms and convolutions

1. Elements of the convolution theory for integral transforms with hypergeometric type kernels: In this talk we exhibit an approach to construct convolutions for the Mellin type transforms developed by the author in the early 90th. The method is based on the theory of double Mellin-Barnes integrals. Some properties of the convolutions and several examples are demonstrated.

2. An introduction to the theory of the index transforms: In this talk we introduce a class of non-convolution integral transformations of the Kontorovich-Lebedev type where the integeration is realized by the parameter (index) of the hypergeometric functions as the kernel. Mapping properties, boundedness conditions and parseval equalities are established. Certain examples of the reciprocal formulas are exhibited.

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